DARDEN BUSINESS PUBLISHING
UNIVERSITY of VIRGINIA

UV3962

DIVERSIFICATION

Don't put all your eggs in one basket.
—Anonymous

Diversification is perhaps the deepest idea in finance. Put simply, when multiple risky investments are combined in a portfolio, the portfolio returns are less risky than the individual assets. Intuitively, one of the investments may have good returns while another investment does poorly and these two effects will offset each other, leaving the combined investment less risky. Because investors dislike risk, being able to diversify away a portion of the risk simply by investing in a number of stocks is an exciting prospect.

The notion of diversification is easiest to understand if we consider the case of a portfolio of just two stocks, Stock A and Stock B. All the concepts discussed in this note extend to portfolios made up of more than two investments, but such an analysis would be unnecessarily complicated.

Portfolio Weights

A good place to start when describing a portfolio is the proportions invested in the various components, which are called weights. The weights simply tell us what fraction of our investment dollar is invested in each stock. If our portfolio were \$100 in Stock A and \$100 in Stock B, then the weight in Stock A (denoted w_A) would be 0.5, because we have \$100 of our \$200 portfolio invested in Stock A.

The weight in Stock B could be calculated the same way. Alternatively, we could say that we are investing in only two assets and the weights must add up to 1, so the weight in Stock B is given by $(1 - w_A)$.

This note was prepared by Jonathan F. Spitzer, Assistant Professor of Business Administration. It was written as a basis for class discussion rather than to illustrate effective or ineffective handling of an administrative situation.

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Portfolio Returns

To understand the expected return on a portfolio, we need to know the expected return on its components. We can denote the expected return on Stock A as $E(R_A)$ and the expected return on Stock B as $E(R_B)$.

Calculating the expected return on a portfolio is elegantly simple. Denoting the expected return on a portfolio made up of Stocks A and B as $E(R_P)$, then:

$$E(R_P) = w_A E(R_A) + (1 - w_A) E(R_B) \tag{1}$$

The expected return on a portfolio is the weighted average expected returns on the stocks in the portfolio. For instance, if $w_A = 0.5$, $E(R_A) = 8\%$, and $E(R_B) = 12\%$, then the expected return on the portfolio, $E(R_P)$, is $0.5 \times 8\% + (1 - 0.5) \times 12\% = 10\%$.

Portfolio Variance

Although we generally consider the standard deviation of returns, the math is done in terms of the variance. Because the standard deviation is merely the square root of the variance, there is no problem converting from one to the other.

We can denote the variance of the return on Stock A as σ_A^2 and the variance of the return on Stock B as σ_B^2.[1] We can also denote the correlation between the returns on Stock A and Stock B as ρ_{AB}. With all these inputs, we can calculate the variance of the portfolio returns σ_P^2 as follows:

$$\sigma_P^2 = w_A^2 \sigma_A^2 + (1 - w_A)^2 \sigma_B^2 + 2 w_A (1 - w_A) \rho_{AB} \sigma_A \sigma_B \tag{2}$$

The equation may seem daunting at first, but like a good meal, it is more enjoyable when broken down into bite-size pieces. There are three pieces. The first piece, $w_A^2 \times \sigma_A^2$, reflects the amount we have invested in Stock A as well as the variance of the return on Stock A. In a similar fashion, the second term, $(1 - w_A)^2 \sigma_B^2$, reflects the amount we have invested in Stock B and the variance of the return on Stock B. The really interesting term is the third, $2 w_A (1 - w_A) \rho_{AB} \sigma_A \sigma_B$, which reflects the diversification benefit.[2]

[1] Because the standard deviation is the square root of the variance, the standard deviation of the return on Stock A will be σ_A and the standard deviation of the return on Stock B will be σ_B.

[2] The "2" arises because we have to account for the relationship between A and B as well as between B and A, which will be the same.

[redacted]

Let us first consider the situation where the returns on the two stocks are perfectly correlated ($\rho_{AB} = 1$). If the returns on the two stocks are perfectly correlated, then when one stock has good returns, the other stock will also have good returns (and vice versa), so there is no way for the returns to offset each other and thus no diversification benefit. When $\rho_{AB} = 1$, equation (2) can be rewritten[3] as:

$$\sigma_P^2 = \left(w_A \sigma_A + (1 - w_A)\sigma_B\right)^2 \tag{3}$$

Taking the square root of both sides, we find:

$$\sigma_P = w_A \sigma_A + w_B \sigma_B$$

It becomes clear that there is no diversification benefit—the portfolio's standard deviation is just the weighted average of the standard deviations of the two stocks.

The magic happens when the correlation is less than perfect. Going back to the third term of equation (2), let us consider what happens when we reduce the correlation from 1 while keeping all the weights and variances constant. As we reduce the correlation, the value of the third term will also decrease, which will in turn reduce the portfolio variance. This reduction in portfolio variance is the diversification benefit.

There is a key point here, one that is often misunderstood. Even if the correlation is 0.9, there is still a gain from diversification. When the correlation is 0.9, almost all the good returns on the first stock will coincide with the good returns on the second stock, but a very small fraction will not. This fraction of "different" returns provides a very small amount of "canceling out," which does reduce the overall portfolio variance. Students often state that the investments need to be uncorrelated (i.e., $\rho_{AB} = 0$) for diversification to exist, but there are actually diversification benefits whenever the correlation is less than perfect (i.e., $\rho_{AB} < 0$).

Let us consider an equally weighted portfolio of Stock A and Stock B (i.e., $w_A = 0.5$). The variance of Stock A is 25% and the variance of Stock B is 36%. Let us first consider the case where $\rho_{AB} = 1$. We will use equation (2) to calculate the portfolio variance:

$$\sigma_P^2 = 0.5^2 \times 25 + (1 - 0.5)^2 \times 35 + 0.5 \times (1 - 0.5) \times 1 \times \sqrt{25} \times \sqrt{36} = 30.25\%$$

The standard deviation is $\sqrt{30.25} = 5.5\%$. Note that, as advertised, this is the weighted average of the standard deviations of the two stocks, which is given by $0.5 \times \sqrt{25} + (1 - 0.5) \times \sqrt{36}$.

[3] If you really enjoy algebra, expand equation (3) to show that it is the same as equation (2) with $\rho_{AB} = 1$.

[redacted]

Now let us consider the situation where $\rho_{AB} = 0.9$. We can calculate the portfolio variance as follows:

$$\sigma_P^2 = 0.5^2 \times 25 + (1-0.5)^2 \times 35 + 0.5 \times (1-0.5) \times 0.9 \times \sqrt{25} \times \sqrt{36} = 28.75\%$$

The standard deviation is $\sqrt{28.75} = 5.36\%$, which is lower than the 5.5% when there is perfect correlation, so there are diversification benefits even when the correlation is as high as 0.9.

Table 1. Portfolio risk for various levels of correlation.

ρ_{AB}	σ_P^2	σ_P
1	30.25%	5.50%
0.9	28.75%	5.36%
0.5	22.75%	4.77%
0	15.25%	3.91%
−0.5	7.75%	2.78%
−0.9	1.75%	1.32%
−1	0.25%	0.50%

Table 1 contains portfolio variances and standard deviations for a number of different correlations. Clearly, the portfolio's standard deviation drops considerably as the correlation decreases from perfect positive correlation. The stocks traded in the market generally do not have returns that are perfectly correlated with each other, so there is always a diversification benefit that allows investors to reduce the overall portfolio risk of their investments.

Financial economists are fond of saying that there is no such thing as a free lunch to make the point that everything has a cost. Yet diversification represents the one true "free lunch" available. Portfolio returns will always be the weighted average of the returns on the investments, but so long as the correlation is less than perfect, the portfolio risk (as measured by the standard deviation of portfolio returns) will be *less* than the weighted average of the risks of the individual investments.